Charged vector mesons in a strong magnetic field

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We show that charged vector mesons cannot be condensed by a magnetic field. Although some hadron models predict the charged vector meson condensation in a strong magnetic field, we prove, by means of the Vafa-Witten theorem, that this is not the case in QCD. We also perform the numerical analysis for the meson mass and condensation in lattice QCD. The lattice QCD data confirm no charged vector meson condensation in a magnetic field.

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I. INTRODUCTION

Strong magnetic fields drastically affect dynamics of the strong interaction. Such strong magnetic fields are created in the early universe, heavy-ion collision experiments, and magnetars [1–5]. A magnetic field modifies energy spectrum of particles, in particular, charged particles. The energy levels of charged particles are quantized, which is the so-called Landau quantization, due to circular motion by the Lorentz force. In addition to the Landau quantization, the anomalous Zeeman splitting happens for the particles with spin. The energy levels of free charged particles in a background magnetic field parallel to z axis are given as $E^2 = p_z^2+(2n+1)|qB|-gs_zqB+m^2$, where g is the g-factor. Masses of charged hadrons are expected to obey this formula in the weak magnetic field limit. For example, charged pions become heavier in a magnetic field as $m^2_{\pi^\pm}(B) = m^2_{\pi^\pm}(B = 0) + eB$. On the other hand, the polarized charged ρ mesons become lighter as $m^2_{\rho^\pm}(B) = m^2_{\rho^\pm}(B = 0) - eB$, where the g-factor is estimated as $g = 2$ [6–11]. Intuitively, the mass of charged ρ mesons becomes zero at the critical magnetic field $eB_c \approx m^2_{\rho^\pm}(B = 0)$, and seems imaginary above the critical magnetic field. The charged vector meson condensation above the critical magnetic field was suggested in hadronic models [12, 13]. This condensation was also discussed in more microscopic theories, such as the extended Nambu–Jona-Lasinio model (NJL) [14], the lattice QCD simulation [15], and models of the gauge/gravity correspondence [16, 17].

In this paper, we point out that the charged vector meson condensation cannot occur in QCD in a strong magnetic field. More generally, any global-internal symmetry is not spontaneously broken by a magnetic field. For the vanishing magnetic field, this is known as the Vafa-Witten theorem [18], which is a consequence of the positivity of the fermion determinant. The positivity maintains even in the existence of a magnetic field, so that the Vafa-Witten theorem works.

We also study how the charged vector meson mass depends on a magnetic field in lattice QCD. Meson masses in a magnetic field have been calculated in a few cases [11, 19, 20]. When the strength of a magnetic field exceeds the QCD scale, internal structures of hadrons are important. Lattice QCD is the best way to study hadron properties in a strong magnetic field quantitatively.

This paper is organized as follows: In Sec. II, we discuss the possibility of the charged vector meson condensation, and analytically show that the charged vector meson condensation does not occur in a magnetic field. In Sec. III, we evaluate the meson masses in a magnetic field using lattice QCD, and numerically confirm the condensation does not occur. Section IV is devoted to a summary and discussion.

II. THEORETICAL ANALYSIS

A. Symmetry of QCD in a magnetic field

We consider two-flavor QCD in a magnetic field. The up and down quark masses are the same and nonzero, $m \equiv m_u = m_d \neq 0$. We assume that the strong CP angle is zero. We work in Euclidean space, and choose $\gamma_\mu^\dagger = \gamma_\mu$ and $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$. The Lagrangian reads

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}G_{\mu\nu}G_{\mu\nu} + \bar{\psi}(\not{D} + m)\psi, \tag{1}$$

with the QCD field strength $G_{\mu\nu} = -i[D_\mu, D_\nu]/g_{\mathrm{YM}}$. The Dirac operator is

$$D_\mu = \partial_\mu - ig_{\mathrm{YM}}A_\mu - iqA^{\mathrm{em}}_\mu \tag{2}$$

with the electric charge $q = e(\tau_3 + 1/6)$.

When $A^{\mathrm{em}}_\mu = 0$ for all μ, this Lagrangian has an internal global $SU(2)_V \times U(1)_B$ symmetry, in addition to space-time (Poincare) and discrete (C, P, and T) symmetries. Chiral symmetry is explicitly broken by the quark mass.

When a constant external magnetic field exists along the z-direction, parts of these symmetries are explicitly broken. The global symmetry is broken down into $U(1)_B \times U(1)_{I_3}$. C and T is broken while P is preserved. Lorentz symmetry is also broken down into $SO(1, 1)_{t,z} \times SO(2)_{x,y}$.

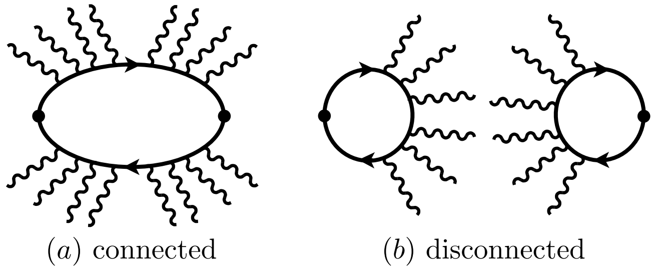

FIG. 1: Typical Feynman diagram for a two-point function in a given background field.

B. Vafa-Witten theorem

Here we discuss the possibility of the charged vector meson condensation in a magnetic field following Weinberg's textbook [21]. The order parameters of the charged vector meson condensation are $\langle\bar\psi\tau_+\gamma_+\psi\rangle$ and $\langle\bar\psi\tau_-\gamma_-\psi\rangle$ with $\gamma_\pm \equiv (\gamma_1 \pm i\gamma_2)/2$ and $\tau_\pm \equiv (\tau_1 \pm i\tau_2)/2$. We will show that this type of the operator cannot condense in QCD (more generally, vector-like gauge theories). In the vanishing magnetic field, this theorem is called the Vafa-Witten theorem [18].

Since the Dirac operator in Euclidean space is anti-Hermitian, the eigenvalues of the Dirac operator are pure imaginary. In addition, the Dirac operator anticommutes with γ_5, which implies that the Dirac operator has a pair of eigenvalues $\pm i\lambda$ for nonzero λ. This can be shown in the following: Suppose that ψ_λ is the eigenstate of the Dirac operator with an eigenvalue $i\lambda$ (λ is real), which satisfies the eigenvalue equation, $\not{D}\psi_\lambda = i\lambda\psi_\lambda$. Acting $-\gamma_5$ to the both sides of the eigenvalue equation, one obtains $\not{D}\gamma_5\psi_\lambda = -i\lambda\gamma_5\psi_\lambda$. Therefore, $\gamma_5\psi_\lambda$ is the eigenstate with an eigenvalue $-i\lambda$. This fact implies that the fermion determinant is real and positive:

$$
\begin{aligned}
\det(\not{D}+m) &= \prod_\lambda (i\lambda + m) \\
&= m^{n_0}\prod_{\lambda>0}(i\lambda+m)(-i\lambda+m) \\
&= m^{n_0}\prod_{\lambda>0}(\lambda^2+m^2) > 0 \ ,
\end{aligned}
\tag{3}
$$

where n_0 is the number of zero eigenvalues of the Dirac operator. By integrating the fermion degrees of freedom, the effective measure is obtained as

$$
d\mu = \prod_{\mu,a,x} dA_\mu^a(x)\det(\not{D}+m)e^{-S[A]} \ , \tag{4}
$$

which is also real and positive. Here $S[A]$ is the action of the gauge field. We define the gauge average with the effective measure as

$$
\langle\mathcal{O}\rangle_A \equiv \frac{1}{\int d\mu}\int d\mu\ \mathcal{O} \ . \tag{5}
$$

Since the effective measure is real and positive, $|\langle\mathcal{O}\rangle_A| \le \langle|\mathcal{O}|\rangle_A$, which plays important role in the QCD inequalities.

Another important property of the Dirac operator is that the following operator is bounded by the bare quark mass:

$$
\begin{aligned}
\frac{1}{\not{D}+m}\frac{1}{\not{D}^\dagger+m} &= \frac{1}{-\not{D}^2+m^2} \\
&= \sum_\lambda \frac{1}{\lambda^2+m^2}|\lambda\rangle\langle\lambda| \\
&\le \sum_\lambda \frac{1}{m^2}|\lambda\rangle\langle\lambda| = \frac{1}{m^2} \ ,
\end{aligned}
\tag{6}
$$

which is independent of the gauge field.

Now, let us consider the symmetry breaking. We write the order parameter as

$$
\phi \equiv \frac{1}{4VN_cN_f}\int d^4x\bar\psi(x)F\psi(x) \ , \tag{7}
$$

with $N_c = 3$, $N_f = 2$, and V being the space-time volume. The operator F depends on the isospin and spinor, which is normalized as $\mathrm{Tr}FF^\dagger = 1$. Here, Tr denotes sum over space-time, isospin, and spinor indices; and is normalized unity, $\mathrm{Tr}\mathbf{1} = 1$. In order to take into account the possibility of an inhomogeneous phase, F may depend on space-time coordinate. For example, if one considers an charge density wave with a single-plane wave, $\langle\psi^\dagger\psi\rangle = \Delta\cos\theta(x)$, one may choose $F = \sqrt{2}\gamma^0\cos\theta(x)$; then $\langle\phi\rangle = \Delta/(4N_cN_f\sqrt{2})$. For the charged vector meson condensation, we can choose $F = \tau_+\gamma_+f(x)$, where $f(x)$ is the function characterizing inhomogeneity.

In the following, we consider the order parameter satisfying

$$
\mathrm{Tr}\frac{1}{\not{D}+m}F = 0 \ . \tag{8}
$$

This is the case with the condensation of the charged-vector meson since it carries the electric charge and its trace vanishes. It cannot, however, be concluded that the symmetry breaking does not occur because this vacuum might be unstable for a small disturbance. To make the discussion of the symmetry breaking precise, we need to add an explicit breaking term $\epsilon\bar\psi\mathit{\Gamma}\psi$ into the Lagrangian, and we take $\epsilon \to 0$ after all the averages. Here, $\mathit{\Gamma}$ depends on isospins, spinors, and space-time coordinate, and is normalized as $\mathrm{Tr}\mathit{\Gamma}\mathit{\Gamma}^\dagger = 1$.

Expanding $\langle\phi\rangle$ in terms of ϵ, we have

$$
\begin{aligned}
\langle\phi\rangle &= \epsilon\langle\mathrm{Tr}\frac{1}{\not{D}+m}\mathit{\Gamma}\frac{1}{\not{D}+m}F\rangle_A \\
&\quad + \epsilon\langle\mathrm{Tr}\frac{1}{\not{D}+m}\mathit{\Gamma}\ \mathrm{Tr}\frac{1}{\not{D}+m}F\rangle_A + \mathcal{O}(\epsilon^2) \ .
\end{aligned}
\tag{9}
$$

The diagrammatic representation is shown in Fig. 1. The second term in the right hand side of Eq. (9) vanishes due

to Eq. (8). Let us consider the absolute value of the first term in the right hand side,

$$\begin{aligned}
&\epsilon \left| \mathrm{Tr} \frac{1}{\not{D}+m} \Gamma \frac{1}{\not{D}+m} F \right| \\
&\le \epsilon \sqrt{\mathrm{Tr} \frac{1}{\not{D}^\dagger+m} \frac{1}{\not{D}+m} \Gamma\Gamma^\dagger \mathrm{Tr} \frac{1}{\not{D}^\dagger+m} \frac{1}{\not{D}+m} F F^\dagger} \\
&\le \frac{\epsilon}{m^2} \sqrt{\mathrm{Tr}\Gamma\Gamma^\dagger \mathrm{Tr} F F^\dagger} = \frac{\epsilon}{m^2} \ ,
\end{aligned} \tag{10}$$

where we used Eq. (6). This is bounded by ϵ/m^2. Similarly the higher order terms are bounded by the factor $(\epsilon/m)^n/m$. This bound is independent of the gauge configuration, so that it does not change by the average of the gauge field. Then, $\langle\phi\rangle$ is an analytic function of ϵ and has no singularity as long as the quark mass is nonzero, so that we can smoothly take $\epsilon \to 0$. In this limit, $\langle\phi\rangle$ vanishes; therefore, the charged vector meson condensation does not occur in QCD.

More generally, according to Vafa-Witten's argument [18], one can show that no Nambu-Goldstone boson exists in a magnetic field if $m \neq 0$. In other words, any internal symmetry cannot be spontaneously broken down in a magnetic field with a nonzero fermion mass as long as (discrete) translational symmetry is not broken at least in one direction.

We note that this argument cannot apply to the vector-like theory in which the vector field, V_μ^a, has an electric charge, like the extended NJL model [22, 23]. In this case, the given background carries the electric charge, so that

$$\mathrm{Tr} \frac{1}{\not{D}+m} F \tag{11}$$

may not vanish, where $D_\mu = \partial_\mu - i\tau^a V_\mu^a - iqA_\mu^{\mathrm{em}}$. The disconnected diagram shown in Fig. 1(b) contributes to the condensate; therefore the previous argument cannot apply, and the charged vector meson condensation cannot be excluded. In fact, the charged vector meson condensation is found in the extended NJL model [14].

C. QCD inequality

In the previous subsection, we discussed that the charged vector meson condensation does not occur. Here we consider the lower bound of the charged vector meson mass. For this purpose, we apply the so-called the QCD inequalities [24–28], which follows

$$\begin{aligned}
&\langle \rho^-(x)\rho^+(y) \rangle \\
&= -\langle \mathrm{tr} S_u(x,y)\gamma_+ S_d(y,x)\gamma_- \rangle_A \\
&\le \sqrt{\langle \mathrm{tr} S_u(x,y) S_u^\dagger(x,y)\rangle_A \langle \mathrm{tr} S_d(x,y) S_d^\dagger(x,y)\rangle_A} \ ,
\end{aligned} \tag{12}$$

where we have used the Cauchy-Schwartz inequality in the second line. Here $S_u(x,y)$ and $S_d(x,y)$ are the propagators of up and down quarks, respectively. Thanks to γ_5 hermiticity of the Dirac operator, $\not{D}^\dagger = \gamma_5 \not{D} \gamma_5$, it follows

$$\begin{aligned}
&\langle \mathrm{tr} S_u(x,y) S_u^\dagger(x,y) \rangle_A \\
&= \langle \mathrm{tr} S_u(x,y)\gamma_5 S_u(y,x)\gamma_5 \rangle_A \\
&= \langle \bar{u}(x) i\gamma_5 u(x) \bar{u}(y) i\gamma_5 u(y) \rangle_{\mathrm{conn.}} \ .
\end{aligned} \tag{13}$$

We will call this correlation function in the last line the "connected" neutral pion, π_u^c (and π_d^c for the down quark). At large distance, $\langle \rho^-(x)\rho^+(y)\rangle \sim \exp(-|x-y|m_{\rho^+}) \le \exp(-|x-y|(m_{\pi_u^c}+m_{\pi_d^c})/2)$. Therefore, $m_{\rho^\pm} \ge (m_{\pi_u^c}+m_{\pi_d^c})/2 \ge \min(m_{\pi_u^c}, m_{\pi_d^c})$. Although the connected neutral pions are not physical neutral pions, they can be calculated in the lattice QCD, as discussed in the next section.

III. NUMERICAL ANALYSIS

A. Simulation setups

We performed the quenched QCD simulation with $\beta = 5.9$. For the quark propagator, we used the Wilson fermion with the hopping parameter $\kappa = 0.1583$. These parameters correspond to the lattice spacing $a \simeq 0.10$ fm and the meson mass ratio $m_\pi/m_\rho \simeq 0.59$ [29]. A constant Abelian magnetic field is applied in the z-direction [30]. The masses of up and down quarks are the same, but the electric charges are different as $q = \mathrm{diag}(q_u, q_d) = \mathrm{diag}(2e/3, -e/3)$. To suppress unnecessary contributions from finite-momentum excited states, all the correlation functions were projected onto $p_z = 0$ by averaging the positions of the source operators in the z-direction. Unlike the usual meson mass calculation without the magnetic field, the correlation functions were not projected onto $p_x = p_y = 0$ because the background Abelian gauge field breaks translational invariance in the x- and y-directions.

We calculated the correlation functions

$$G_X(t_1 - t_2) = \langle X^\dagger(\vec{x}, t_2) X(\vec{x}, t_1) \rangle \tag{14}$$

of four mesons $X = \{\pi^+, \rho^+, \pi^0, \rho^0\}$. For charged and neutral ρ mesons, we used the correlation function

$$G_\rho(t_1 - t_2) = \frac{1}{2} \sum_{\mu=1,2} \langle \rho_\mu^\dagger(\vec{x}, t_2) \rho_\mu(\vec{x}, t_1) \rangle \tag{15}$$

with $\rho_\mu^+ = \bar{\psi}\gamma_\mu\tau_+\psi$ and $\rho_\mu^0 = \bar{\psi}\gamma_\mu\tau_3\psi$. This correlation function couples to both of the polarized and anti-polarized (i.e., $s_z = \pm 1$) components of ρ mesons. Only the lowest energy state survives in the large t limit. Even if we do not know which component is the lowest energy state, especially of the ρ^0 meson, we can automatically extract the mass of the lowest energy state from this correlation function.

We did not calculate the $\mu = 3$ (i.e., $s_z = 0$) components of ρ mesons. It is too difficult to calculate them in lattice QCD. In the background magnetic field, the π-ρ_3

mixing exists even for in the connected diagram. Thus, the $\mu = 3$ component of a ρ meson is an excited state of a pion. At least in the weak magnetic field limit, there are a large number of magnetic-splitting states of the pion below the energy level of the ρ-meson state. We cannot calculate such a highly excited state in the lattice QCD simulation.

For neutral π and ρ mesons, we calculated only the connected diagram, which is necessary for the QCD inequality. While the disconnected diagram is forbidden in the absence of the magnetic field, it is allowed in the presence of the magnetic field because the magnetic field breaks isospin symmetry. We ignored the disconnected diagram in this simulation. In this sense, our neutral mesons are not physical ones.

B. Meson masses

We performed the standard mass analysis of ground-state mesons in lattice QCD. The meson masses were extracted from the fitting function

$$G_X(t) = A_X \cosh[m_X(t - aN_t/2)] \tag{16}$$

in large t. The lattice volume is $N_s^3 \times N_t = 16^3 \times 32$. The numerical results are shown in Fig. 2.

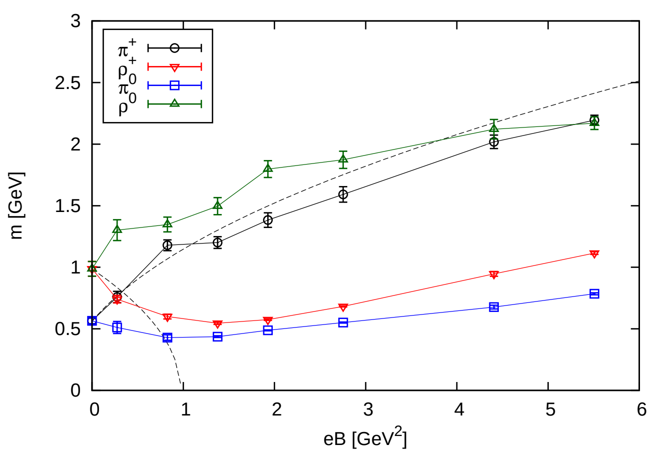

FIG. 2: The meson masses in a magnetic field. The broken curves are $m_{\pi^+}^2(B) = m_{\pi^+}^2(B = 0) + eB$ and $m_{\rho^+}^2(B) = m_{\rho^+}^2(B = 0) - eB$.

The charged pion mass increases in the magnetic field. This mass shift can be explained by the naive mass formula $m_{\pi^+}^2(B) = m_{\pi^+}^2(B = 0) + eB$. As shown in the figure, this formula well reproduces the present lattice result in a weak magnetic field. This behavior was also observed in the full QCD simulation [19]. The lattice data slightly deviate from this formula in a strong magnetic field.

The charged ρ meson mass shows a nontrivial dependence on the magnetic field. When the magnetic field is weak, the mass is a decreasing function of the magnetic field. The naive mass formula, $m_{\rho^+}^2(B) = m_{\rho^+}^2(B = 0) - eB$, reproduces the lattice data. At $eB \simeq 1$ GeV^2, the mass has a nonzero minimum. When the magnetic field is stronger than this value, the mass becomes an increasing function of the magnetic field. As a consequence, the charged ρ meson is always massive and heavier than the connected neutral pion in the whole range of the magnetic field. Although the Wilson fermion does not have the exact positivity, the present lattice result is consistent with the Vafa-Witten theorem and the QCD inequality.

The neutral mesons are much more nontrivial. In the naive mass formula, neutral particles are independent of a magnetic field. The lattice result suggests, however, that the neutral meson masses depend on the magnetic field. This is due to the internal structure of the mesons. To know how the physical neutral mesons behaves in a magnetic field, we have to take into account the disconnected diagram.

When the magnetic field is extremely strong, i.e., $eB \gg 1$ GeV^2, the masses of all the mesons monotonically increases. This is interpreted as a sign that the internal quarks obtain the large magnetic-induced mass. The underlying mechanism is unknown in the present analysis.

C. Meson condensations

To exclude the possibility of the charged ρ meson condensation in lattice QCD, we performed another analysis. If a meson condensation exists, the ground state becomes massless and a long-range correlation appears. The correlation function becomes

$$G'_X(t) = A_X \cosh[m_X(t - aN_t/2)] + C_X \tag{17}$$

in large t. If the constant parameter C_X is finite, C_X corresponds to the squared meson condensation $\langle X \rangle^2$ and m_X corresponds to the mass of the first excited state. A similar analysis was performed in a previous work [15]. However, such a constant term can be easily generated by a finite-volume artifact. We must carefully check the finite-volume artifact.

We calculated the correlation functions $G_X(t)$ with three lattice volumes $N_s^3 \times N_t = 16^3 \times 32$, $20^3 \times 40$ and $24^3 \times 48$, and fitted the results with Eq. (17). In Fig. 3, we show C_X as a function of the lattice volume $V = a^4 N_s^3 N_t$. The magnetic field is fixed at a large value $eB \simeq 4.3$ GeV^2. In a small volume, C_{π^0} and C_{ρ^+} seem finite. In the infinite volume limit, however, all C_X approach to zero. In particular, C_{ρ^+} is zero within the statistical error. From this analysis, we conclude that the charged ρ meson is not condensed by a magnetic field.

As shown in Fig. 3, C_{π^0} is large compared to other mesons. This is an expected behavior because the connected neutral pion is the lightest particle and the finite-volume artifact is the largest for the lightest particle. If

there were the physical charged ρ meson condensation, C_{ρ^+} would be finite and larger than C_{π^0}. To estimate the finite-volume artifact of a small quantity, it is important to compare it with the other quantity which is the most sensitive to the finite-volume artifact.

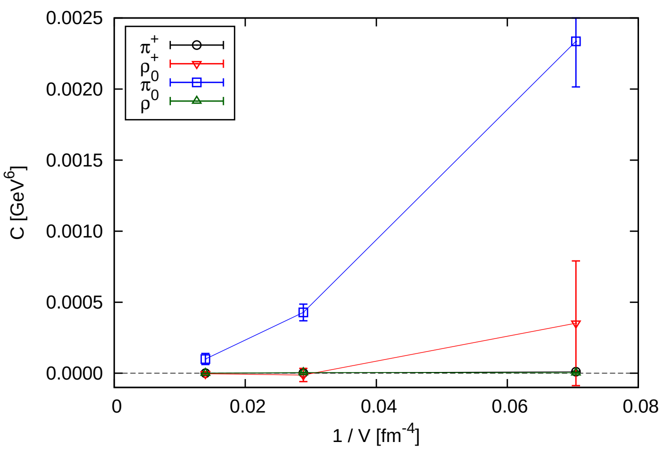

FIG. 3: The volume dependence of the constant parameter C_X in Eq. (17). The magnetic field is $eB \simeq 4.3$ GeV^2.

IV. SUMMARY AND DISCUSSION

We have analytically and numerically shown that charged vector mesons are not condensed by a magnetic field. In general, a magnetic field cannot induce any condensation which breaks global-internal symmetry in QCD. In other words, a strong magnetic field cannot change the QCD vacuum structure. For example, the insulator-superconductor phase transition cannot be induced by a strong magnetic field alone.

The Vafa-Witten theorem works not only in the magnetic field but also at finite temperature, T, so that we can conclude that no Nambu-Goldstone phase associated with spontaneous breaking of global-internal symmetries exists in the phase diagram of (T, B) plane. We note that we cannot exclude the possibility of spontaneous breaking of space-time symmetries because the order parameter does not satisfy Eq. (8), i.e., the disconnected diagram exists.

As expected in hadron models, if a charged vector meson were a point particle, its mass would decrease in a magnetic field. The lattice QCD results support this scenario in the weak magnetic field limit. The mass of the charged vector meson turns to increase at $eB \sim 1$ GeV^2, which is the QCD scale, before the mass reaches zero. When the magnetic field is stronger than this scale, other meson masses also increase monotonically, where the internal structure of the mesons becomes non-negligible and the validity of hadron models break down. Therefore, we expect that some transition occurs from a hadronic phase to a phase governed by the magnetic scale although this is not a phase transition separated by some symmetries. A possible scenario to explain the increasing masses is that the constituent quark mass increases by the magnetic catalysis [31–35]. In this case, the meson masses increase as $\sqrt{eB}$. The detailed analysis for this transition is beyond our scope in this paper.

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